



**The Philosophy Foundation**  
*thinking changes*

## **MATHS AS ENQUIRY**

**The maths group as an enquiry,  
problem solving team for KS2**

## Objectives

The Maths as Enquiry fosters a problem-solving approach in the maths classroom. We developed the package with classroom teachers in response to challenges faced when teaching maths, which include the following:

- Lack of independent thinking among the children
- Little ability to justify or explain the strategies they use
- Children not systematic or experimental enough in tackling problems
- Existing knowledge is not effectively used to solve unfamiliar problems
- Difficulty in turning verbal or real-life problems into number problems correctly
- Children focused too much on the right answer, and not enough on the right way to get there
- Disengagement from higher or lower ability children if the lesson is not pitched at their level

By the end of this series of lessons, the school should see the following changes:

- Greater independence of thought, and value placed on independence
- Children try hard to explain why they have done what they have done, and critically evaluate it, with some success
- More systematic ways of working: note-taking, eliminating disproved alternatives, moderating strategies in the light of results
- Children attempt to use existing knowledge to solve new problems, and assess the success of this
- More success in translating verbal/practical questions into number problems
- Children show more pride in having worked at a problem, made progress and altered their strategy – not only in getting the final answer
- Lower ability children experience the value of perseverance, and more able ones learn that patience is more important than an instant fix

## What We Do

The Philosophy Foundation specialises in running philosophical enquiries in schools that deepen children's thinking, resulting in better reasoning, independent thought, and more precise understanding of concepts. Our whole-class enquiries instill an enthusiasm for collaborative problem-solving that carries over well into the maths curriculum.

The main adjustment we make in the maths classroom is that, while a philosophy question can legitimately lead in a number of different directions (and this is actually true for some maths questions too), in maths there will usually be a single answer to a question and the class will need to be guided to it. However, the spirit of enquiry is the same.

Our ideas have been developed in primary schools with reference to curriculum targets. It is also informed by research and writing on the theory of maths teaching.

## In The School

We run the sessions in partnership with the teachers. When the philosopher is leading, the teacher is encouraged to intervene and interject all the way through.

A thorough explanation of our questioning style is given in the attached paper by Peter Worley, *The Absent Teacher* in **Appendix I**.

The main points are:

- withholding the answer
- children share their thoughts openly
- children become familiar with the point where they 'get stuck' and move beyond it
- minimal introduction – straight into the problem
- experimenting with ideas and learning from the results

We usually run the sessions as one hour of maths per week. They should work equally well in more intensive formats.

Outside of the sessions we run, the teacher can capitalise on the work done with similar questioning and set-up of activities, both in maths and other subject areas.

We recognise that to progress in maths children will need to memorise mathematical facts (such as number bonds or multiplication tables), usually by rote-learning and drilling. Our exploratory style is not meant to replace all other forms of learning, but to provide grounding and motivation.

## Principles & Themes

Some of our ideas will be familiar to teaching professionals already.

The first of these is the use of *low threshold/high ceiling* tasks, which can be attempted by all children but extended for those who are faster. An example would be a sudoku-type puzzle where some children take the whole lesson to complete one puzzle while others finish quickly and can alter the same task to create a new challenge, or begin another similar one.

The aim of this practice is to reach higher and lower ability children at the same time. It encourages perseverance among the lower-ability children, because they are given ample time – the whole lesson – to finish. The activity will not be cut short because other children have finished and need to move on. The higher-ability children often learn perseverance as well, because at first sight they regard the problem as impossible, or needing knowledge they haven't been taught. These same children are also expected to set themselves higher challenges when they do finish rather than rest on their laurels.

We constantly re-iterate the need for the children to think about *strategy*. They need to be able to explain what they are doing and why, as they do it. (We have found it is much easier to get them to do this *during* the process than after it – afterwards they will often say they did what they did 'because it gives you the right answer' and be unable to explain how they know it produces the right answer, and they will only remember successful strategies). We praise children for experimenting with different strategies until they find one that seems to work.

We also often ask *Can you do it another way?* We want them to understand that mathematics all links up, and that there can be various ways of doing or saying the same thing.

As far as possible, we try to get the children to decide for themselves, whether as individuals or a class, what the final answer is, by checking and justification.

Another frequent instruction is *Try it!* We want the children to experiment with different strategies and then assess their success, instead of expecting to be told in advance what will yield the answer.

*Try again!* This is an even more important one. Children need to learn the value of trial and error. Very few of them review their efforts and revise their strategy in the light of the first attempt.

## Maths Enquiry In The Classroom

Each session begins with the whole class on the carpet being presented with a stimulus or challenge. Children share their thinking with each other, and build on their peers' ideas. All children are expected to refer to what other people have said, not just pitch in their own idea. The teacher puts a lot of emphasis on willingness to contribute and experiment, raising this behaviour on a par with getting the final answer, and pointing to the vital role that these contributions and experiments have on the path to success.

In some cases we are able to continue working in this way until the end. But often, the children will need to be more 'hands-on' and try things out individually or in pairs. Some children will also race on to extensions of the problem while others wrestle with the starting question for as long as they need.

Many of the materials are online or on computer, which means that they work well as whole-class activities and at the end, even if we have split up, we aim to reconvene and share.

## Materials

We have sourced the materials from books, websites, and articles, and once teachers have used a selection of them they will know what they are looking for and should be able to source more of their own in a similar vein.

In the course of our research we came to the conclusion that maths problems can be motivating for 2 different reasons. First, they can be genuine practical problems that children can imagine themselves needing to solve in real life. Second, they can be an investigation into numbers, quantities and their inter-relations – a puzzle, basically.

# Maths as Enquiry

It is this second type that we have focused on in our Maths as Enquiry programme. If we are successful, then all the benefits should be transferable to the real-life questions.

The kind of activities that we are looking for are:

- Challenges that need trial-and-error to solve
- Low threshold, high ceiling
- Easy to alter, reproduce, and project on screen
- Based on the basic 4 mathematical operations (+ – x /)
- Fun

The early materials are for problem-solving only, and have very little number content, if any. Then we progress to similar activities but involving number operations that should present few problems. Gradually we move towards Test Base-style questions, but trying to retain the same open-ended approach.

The materials we use are available to the teachers we work with in a handout so that they can continue to use them after we have completed the Maths Enquiry work.

For more information or to book a Philosophy Foundation Maths as Enquiry course in your school please email: [emma@philosophy-foundation.org](mailto:emma@philosophy-foundation.org)

## Appendix I

### The Absent Teacher: Preparing Children For The Real World, by Peter Worley

If you are unfamiliar with the '9 dot problem' (or have forgotten it) then, before reading this article, I would urge you to attempt the puzzle first:

#### *The 9 Dot Puzzle*



Task: cover all 9 dots with four (or less), straight, continuous lines without tracing over the lines. Do not read on until you have had a go. The solution is at the end of the article.

\*\*\*

The teacher walks into the classroom and says 'good morning' to the children of a Year 5 class. He explains that today they will be doing something different...

'I am going to set you some puzzles over the next session or two. Some of them, or all of them, may be unsolvable. I shall not tell you which are solvable and which are not. You are entitled to 'give up' at any time, however, I will only take you to have 'given up' when no one is left in the classroom who still wants to try. If there's even just one person who still wants to try then you will not have 'given up'. I should also explain that if you choose to 'give up' I will not tell you any answers. Is that clear?

'Now for the first puzzle. You will see, on the board, 9 dots that form a square:

#### DIAGRAM OF THE 9 DOTS

'The task is to cover all 9 dots with just four or less, straight, continuous lines. By 'four or less' I mean 'not five or more'. By 'straight' I mean 'not bent or curved in any way'. By 'continuous' I mean that, in principle, you 'shouldn't *have* to take your pen off the board to complete the task'. [It should be noted here that if they do take their pen off the board then that is not automatically a contravention of the rule. Only if they would *have to* in order to complete their 'solution'.]

'I shall not answer any questions. All I shall do is tell you when you break the rules. If you have a question then the only way to have it answered is to come up and try it. I shall tell you if you have broken the rules and I will tell you how you have broken the rules. Other than that I shall say nothing until later on when I may provide a clue or two. But remember if it is unsolvable then the clues won't work. That's for you to decide.

'I encourage you to get up and try things out especially as you will not have any paper to 'try out' on. And remember: everything you each do will help the other members of the class in some way, so please: TRY THINGS OUT.

'Now, who would like to come and start things off?'

\*\*\*

This is the beginning of a lesson plan designed to enable primary school children to attempt the famous '9 dot problem'. I have been attempting to engage children of this age with this problem for the best part of 10 years but have usually found that I end up having to show them how to do it, as they can get so far with the problem but then get stuck. I was convinced that the problem was in the way I was engaging them and not that it was too advanced.

Inspired by some features of the Socratic pedagogical method found in the *Meno* dialogue by Plato, I eventually devised this method, which, so far has yielded a 100% success rate with Years 4 – 8 (ages 8 – 13) children.

The main Socratic principles derived from Plato that inform this approach are as follows:

- The children can recover this knowledge for themselves if given the opportunity.
- The teacher only gives clues when absolutely necessary and draws the clues from the children's own attempts wherever possible.

- There is the presence of uncertainty and the possibility of inconclusiveness (Greek: *aporia*). This is not, I will argue, stultifying but motivating.

One notable absent Socratic principle however is that of *questioning*. Even the use of effective questioning has been removed, or at least minimised, in this unusual session. The ‘teacher’ is as absent as is possible whilst still retaining a guiding, pedagogical role. Here is a list of the salient pedagogical features of this lesson:

- Uncertainty as motivation
- Collective intelligence
- Collaborative learning
- Peer modelling
- Risk taking
- Time to ‘unfold’ the problem
- Creativity
- Overcoming fear
- Innovation
- Experimentation
- Logical, stepwise learning

Before I say something about each of these features let me start by continuing to outline the methods of the teacher during and at the end of the session.

The activity of trying to solve the puzzle is best done with an interactive whiteboard (on the squared paper setting). The reason for this is that it is the easiest way for the contributions to be preserved for all to see. Inevitably, questions will be asked. If this happens the teacher should resist answering them but remind them that they can only answer questions by trying them out. If they do and it clearly breaks a rule then the teacher should thank them and explain how what they have done has helped the class to better understand the rules. Remind them that *every contribution is helpful*. When a set number of contributions have been made (e.g. 5 or 10) the teacher then says that they will provide a clue. The clues, as often as possible should be taken from the examples already on the board. For instance, it is quite common for someone to draw a line that extends beyond the limit of the implied square. This is sometimes done deliberately, and sometimes by mistake. If it is done, then however it is done, this makes a perfect clue. Alternatively, someone may draw something that resembles the ‘kite’ shape that the solution produces, so this too would make a good clue example. Once the clues have been provided it might be advisable to ask each subsequent contributor how they have used the clue. This gentle insistence brings their attention to the

clue information. (Many of the children I have worked with have a habit of seeing the clues but then don't use them.) Another good question to ask is: 'What is unique about the clue example(s)?' or 'What do the clues have that the others don't?' From then on the teacher simply gives a new clue every time a child provides one. Other than that the teacher's main tool is patience. One important principle to bear in mind when running this session is not 'When can I jump in and explain something?' but, instead, 'How can I keep 'out of it' for as long as possible, intervening only when necessary?' I describe this teaching principle as 'absence'.

*Uncertainty as motivation:* Whenever I have run this session the children always respond with a slight measure of shock when faced with the possibility that they will not solve the problem and that they will not be told the answer. The children inhabit a culture of 'back of the book' answers and tell me that they have never had an experience like this one. Very often some children will opt to 'give up' and some become quite aggressive in their insistence on this. However, I have never yet had a session where at least one member of the class did not want to continue. This provides an extremely important learning outcome, during and after the session, for all the children involved and I hope that the experience will stay with them for a long time. I argue that a natural position in the 'real world' is one of uncertainty. When problems need to be solved in real situations it is often far from clear that there is an answer prior to one being found. Children need to have experienced this 'real world' possibility and be psychologically prepared for such an outcome much more than they currently are. Socrates, Plato and Aristotle all argued for *aporia* ('puzzlement', 'perplexity', 'inconclusiveness') as a positive state, motivating in itself to seek for further knowledge or to a solution to the cause of the *aporia*. All too often children experience perplexity as a negative experience, a sign that they have failed or are not clever.

*Collective intelligence, peer modelling and collaborative learning:* All too often children are assessed on their individual achievements but many important individual insights and much individual progress is made in the context of interactive work. From this session you might think that the peer modelling would follow this pattern: where 'the solver' shows everyone else how it's done. But it is quite the opposite. When the solver solves the puzzle the children are psychologically disposed to perceive the solver as 'the hero'. Once the puzzle has been solved the first question to the solver should be: 'Well done! Now, what gave you the idea to do this?' In all cases so far, they have referred to an earlier example provided by another child. Interestingly, the child referred to is very often not a 'gifted' child. The teacher should continue this line of questioning to reveal a connected thread of 'inspirations'. Once the 'thread' has been noticed by the class a

shift occurs from seeing the solver as 'the hero' to seeing the achievement as a collective one.

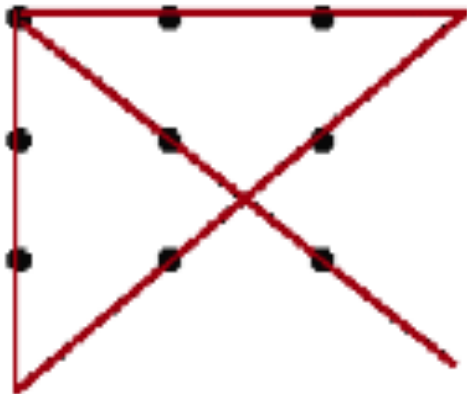
*Risk taking, experimentation, innovation and overcoming fear:* Some of the gifted and talented pupils will often not want to get up and have a go until they feel that they know how to solve it. They will often sit and watch the others get up and try (but 'fail') and then step in at the end when they have seen the solution and swoop to make the victory strokes. An interesting observation from this behaviour is that they are afraid to try and fail at the task. Gifted students are therefore suppressing an experimental instinct and disallowing themselves to benefit from experimentation or develop the 'art of learning from making mistakes' in a social, collaborative context. At the end of the session I do not allow any pupils who refused to have a go to take any of the credit. This may seem a little harsh but I am sure you can see the value of this. The way I phrase it is to say: 'Anyone in the classroom who got up and had a go can claim the credit for having solved what seemed to be an impossible task. Unfortunately, if you did not have a go, then, I'm afraid you can't.' After the session I show them examples from Leonardo da Vinci's notebooks in which he can be seen to make all sorts of attempts that he simply crosses or scribbles out. This is one reason why he was such an innovator. The absence of paper to try out on means that they have to try out in front of each other which encourages experimentation.

*Stepwise learning and the need for time to 'unfold' the problem:* When I began doing this puzzle with classes of children I used to schedule it in at the beginning of a session, so would allocate no more than 25 minutes to the task. However, crucial to the success of this session is allowing the time for the children to slowly work it through. The process of 'scaffolding', that is to say, the inter-related way in which the children build and link their ideas, needs time. And this is just what they don't usually have in a normal teaching context. I often receive the same criticism of the techniques that I teach: *that there simply isn't enough time, with all the demands of the curriculum, to use the approach I advocate.* My response to this is that taking the 'time-saving' route of instruction and 'explaining the answer' is false economy. If a child does not have understanding of a topic or procedure then they will need to ask the teacher again and again for 'the answer' and the teacher will find that they need to reiterate the information many times. In the long run this is not the time-saving route it promised to be. Taking the time at the outset to allow understanding to be self-constructed in the child or children means that less reiteration is needed. The reason why teachers often feel the need to explain the answer themselves is because they are jumping ahead of the point that the children have reached. The solution to this problem is to question the

children in such a way that they move step-by-step towards a deeper understanding. To carry this metaphor a little further, the teacher needs to observe which 'step' they are standing on in order to assess what is needed for them to be able to ascend to the 'next step'. If the teacher is explaining the top step while the children are struggling with the first step then, naturally, the teacher will feel frustrated. This is when the teacher will feel that they need to simply provide the answer and the explanation.

*The solution to the 9 dot puzzle:*

This puzzle perfectly characterises the expression 'think outside the box' as the solution is lateral. There is a self-imposed limit perceived by the person trying to solve the puzzle that it must remain within the square implied by the dots. The solution can only be reached once they have noticed that there is no restriction on the length of the lines:



## Appendix II

### Two lesson plans from The Philosophy Foundation's Maths as Enquiry Course.

#### Is this a big room?

This is a great philosophical question in itself, leading as it does to the fact that something can only be big in relation to something else, (OK, the universe is big, but is it a big universe??) so it depends what you're comparing it with. By the way, you'll find a long tape measure useful, once you get started.

However, if you want to get straight to the maths... You can start by asking 'How big is this room?'. Then see how they think the question can be answered. The obvious first answer will be to measure it, but will they be happy to measure just one dimension? Is two enough? If they do settle for just the length of the room, try getting them to imagine that the question was asked by someone who had never seen the room. Would they have all the information they need about its size, or would more help? Eventually, you should at least get a two-dimensional measurement. Let them go ahead and measure it. Beware, the measuring stages of the lesson can take ages, so sometime it's best to do it yourself, under their direction – but obviously there is some value in them practising this practical skill too.

Next, get them to compare it to another room to which they can get access. Ideally, this would have one dimension longer and one shorter than their classroom, so you end up with something like this:

ROOM A: 6.90 and 5.75

ROOM B: 7.80 and 5.25

For some classes, just looking at the difference between width and length will be enough.

But two more possibilities are: introduce the third dimension of height and measure some other things in three dimensions, or introduce the concept of area ( $7\text{m} \times 5\text{m} = 35$  square metres).

In this session, we are trying to introduce the *necessity* of 2D measurement – and then perhaps the idea of area. We are rooting the concept in a practical problem – we need to know which room is biggest. One way to make this sound real is to say that a school has two classes

and one of them has more children in it. So they want to put the bigger class in the bigger room. If you like, this could be your starting point.

## **How long is this piece of string?**

This one covers two main areas. The first is to get them more familiar with what common measurements *look like*, and estimating realistically. The second is to look at the relationship between cm/metres, decimal places, fractions and percentages. A third could be issues of place value too – 6cm on a 10cm string looks the same as 60cm on a 100cm string) It will work well as a diagnostic tool to show you whether they can deal with questions like:

*What's half a metre?*

*What's 10% of a metre?*

*Is 55 cm more than half a metre?*

*What's 60 cm in metres or millimetres?*

It can also work as a way of visualising percentages, as a metre is divided into hundredths.

Start with a ball of string (or masking tape, clothes line etc). You will also find clothes pegs, or bits of coloured thread/ribbon, useful as a way of marking distances along the line. First get different children to estimate one metre along the line. See if anyone has any way of reckoning it (e.g. it's longer than a leg, it's 5 footballs on top of each other, etc). Then measure it out and cut the string. From there you just ask a lot of questions. Get a selection of answers. Ask for reasons. Then measure. Here are some suggestions:

*Where is half? How many cm is it?*

*Where is a quarter?*

*Where is 10cm?*

*Where is 70cm – nearer the middle or nearer the end?*

*Which things in the room do you think are more/less/the same as 1m?*

*What can you measure with a string that you can't measure with a ruler?*

*Can you measure your height using this 1m string instead of a ruler or tape measure? How close can you get to the right answer?*

*Who has the longest arms in the class – use the string.*

Do a few questions as a class, bringing out the different thinking and different strategies. Then give the children pieces of string to work in groups and bring their answers back. They can work on the same questions or have a different question each.