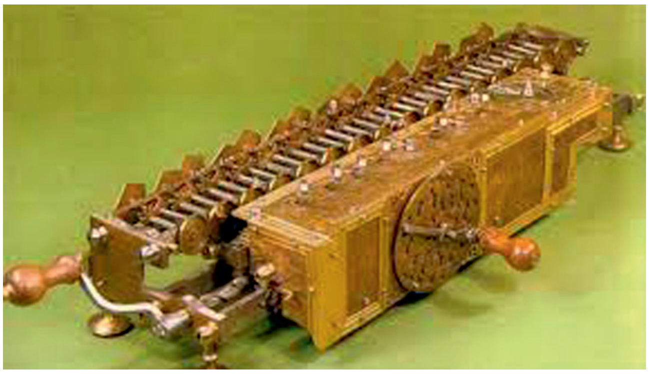
Step Reckoning – what philosophy has to do with mathematical reasoning

Peter Worley highlights the similarities between philosophy and maths

Philosophy and maths? What do this unlikely pair have in common? Well, in four carefully chosen words: reasoning, logic, structure and order.

Around three thousand years ago the Greeks turned their eyes on the world in a new way: they began to abandon the old ways of understanding the world, through myths and legends, and began to construct rational approaches. This was the beginnings of philosophy, which literally means, 'love of wisdom'. At this time 'philosophy' encompassed all disciplines of learning, including mathematics. Thales (c.624–546 BCE) wondered about the natural world's composition, which led to Empedocles's (c.495–430 BCE) theory of the four elements – foreshadowing the periodic table of the nineteenth century. Parmenides (c.485 BCE) invented (or was it discovered?) structured argumentation with premises and conclusions, which led to Zeno's (c.490–430 BCE) many logical paradoxes, as well as to Aristotle's (384–322 BCE) invention of a formal language of logic, paving the way for the modern computer.

The Greeks recognised a direct link between rational philosophy and mathematics. Plato (c.428–347) is said to have had an inscription over the door of his Academy: 'Let no one ignorant of geometry enter.' Years later, the German philosopher Leibniz's (1646–1716), maxim '*Calculemus!*' ('Let us calculate!) reflected his dream, that all problems – be they logical, metaphysical or even *ethical* – would one day be solved through calculation: the Enlightenment dream of reason. Figure 1 is Leibniz's 'step reckoner'(1671), one of the earliest attempts at something resembling a modern computer, well over a hundred years before Babbage.



Leibniz's 'step reckoner'

Perhaps, we might say, that the Greeks' big intellectual mistake was conceiving of science too closely with maths, in other words, as an *a priori* (see 'Armchair philosophy' below) reasoning endeavour, not an experimental one. It is because Aristotle – the giant of medieval philosophy – maintained such a great influence over the minds of that period, and because he had a 'maths-model' of science, that experimental science took such a long time to get off the ground (not really doing so until Redi's famous experiments (1668) falsifying *abiogenesis* – the theory of spontaneous generation – see link in references). In this way, philosophy was *too* much like maths.

More recently, research produced by the Education Endowment Foundation (Gorard 2015 – see link in references), has shown a correlative link between doing 'philosophy for children' and improved performance in maths. One possible reason for this is structural. If children, when doing philosophy are learning – or practising – to structure their thoughts better along similar structural lines to mathematical reasoning, then it may well be this that confers the improvement. Of course, more research needs to be done to establish whether or not this is the cause, but it's certainly plausible enough for it to be one of the next lines of research inquiry.

Philosophy Tools for Maths

What follows is a list of 'critical thinking tools' with accompanying 'key facilitation skills' for each one showing how to use the tools in the classroom. I use all of them for philosophy discussions in order to help structure the conversations in the right way: *logically* and *sequentially*. I have chosen the tools that have the clearest application in and with maths.

1 **Critical thinking tool:** *Modus Ponens* – a very common argument structure that goes like this: *if p then q, p, therefore q*. For example:

Premise 1: If something can provide the answer to a calculation then it can think,

Premise 2: Ceebie (a computer-robot friend) can provide the answer to a calculation,

Conclusion: Therefore Ceebie can think.

This is a well-structured (technical term: *valid*) argument but it's not true (so, technically, not *sound*). The good structure is demonstrated by the fact that *if* the premises (reasons) were true, then the conclusion would *have* to follow. It is shown not to be true, however, by the example of a calculator. A calculator can provide the answer to a calculation, or 'compute', but it cannot *think* because it cannot do any of the other things that might be considered necessary for thinking such as - among others - *deliberate* or *learn* (which includes *making mistakes*, something, incidentally, calculators don't do).

Key Facilitation Skill: *Iffing, anchoring and opening up* - this combination of strategies helps to encourage this kind of structured thinking and expression, implicitly inviting the students to say whether what they said supports their view on the question. So, if the question is 'Can computers think?' then they will either be thinking 'yes', 'no', 'something else' (such as 'yes *and* no' or neither (for instance, if they don't understand the question). If they hold a positive, unqualified thesis (either 'yes' or 'no') then the question is turned into a statement: E.g. 'Yes, I think computers can think.' This is their conclusion. But children don't always say all of this, they might just say, 'Thinking is just calculating.' By *iffing* this and by *anchoring* it to the main question: 'So, if thinking is just calculating, then can computers think?' the student is brought to decide (or *recognise*) what conclusion this brings them to hold (albeit provisionally): E.g. 'Yes.' Then a simple opening up question lets us in to their reasons (or premises): 'Would you like to say why?' 'Because computers calculate so…' The implicit argument here is *modens ponens* move:

- If computers calculate then they can think (because calculating is thinking)
- Computers calculate.
- Therefore computers can think.

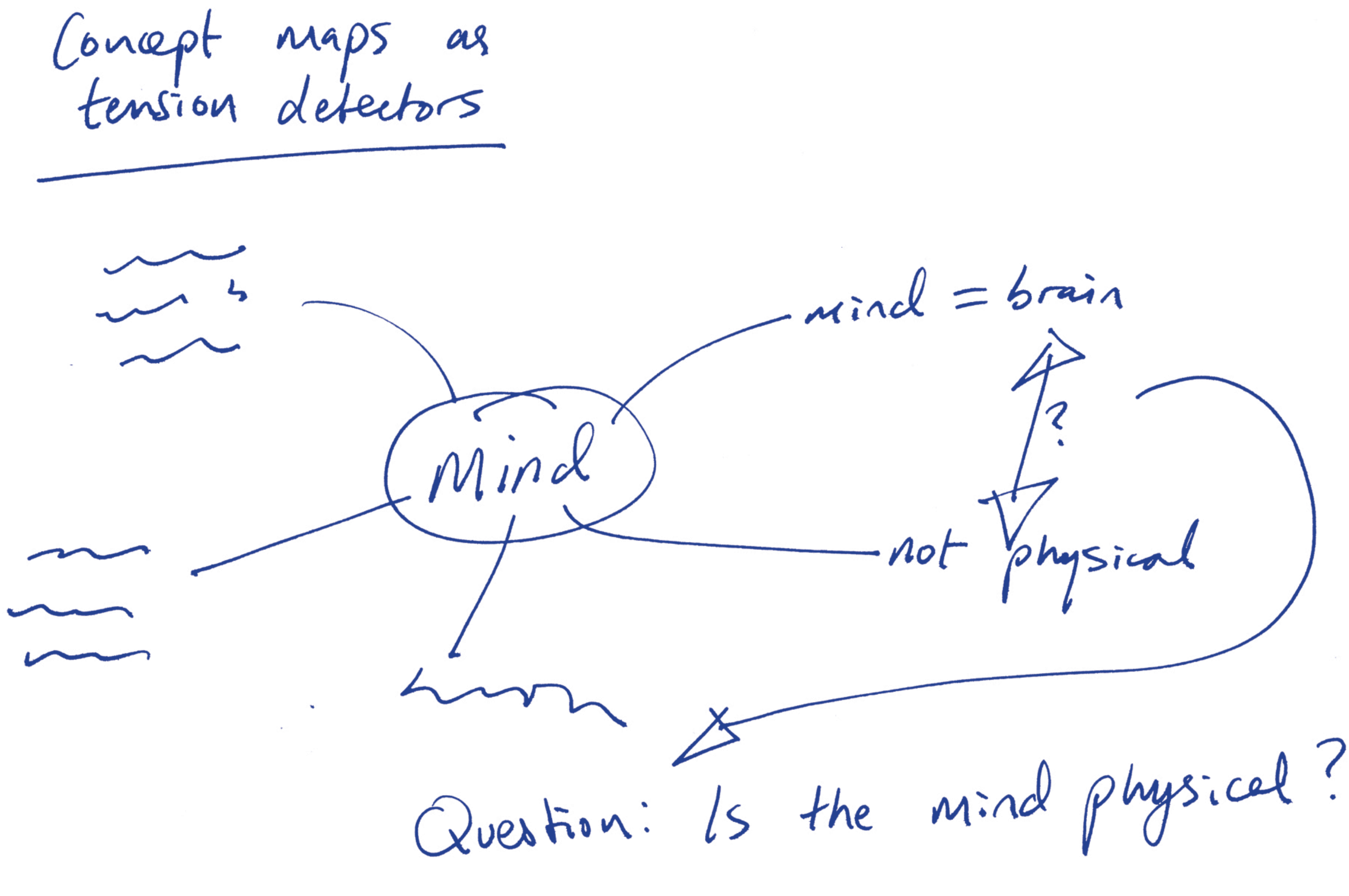
From a logical point of view, this also affords you, the teacher/facilitator, a legitimate opportunity to introduce a possible controversy to the class/group: 'Is calculating the same as thinking?' because, that 'calculating is just thinking' is an assumption that, according to the logical and sequential demands of a philosophical conversation, needs to be questioned.

2 **Critical thinking tool:** *Contradictions and contraries* – in logic a *contradiction* is two statements that can't both be true at the same time or both be false at the same time. For example, the statements 'the ball is all red' and (at the same time) 'the ball is not all red,' lead to a contradiction because if the ball is all red, it's impossible for the ball to not be all red. *Contraries* are two statements that can't both be true but they *can* both be false. For example, the statements 'the ball is all red' and (at the same time) 'the ball is all blue,' are contraries because it has to be either all red or all blue, it can't be both; but it *is* possible for the ball to be another colour altogether; it could be yellow. In common parlance, the word 'contradiction' is used for both of these terms. Logically speaking, if someone uses contradictory or contrary statements then they must be wrong in their reasoning, or expression, at least. This is one of the clear ways (if not the *only* way) in which someone can be categorically wrong when engaged in a philosophical discussion: that is, *if what they say is, or leads to, a contradiction*.

Key facilitation tool: *Tension play: using contradictions and contraries* – it is tempting to smooth out contradictions in children's answers or, for instance, in information gathered on a board about something (such as an unknown number, 'the number must be even' and 'the number must be odd'), but one should resist doing this. Tensions, contradictions and contraries = *learning opportunities*. If someone contradicts themselves then some kind of *response detector* is useful: 'Is there anyone who has something to say about that?' If two children, A and B, give the same reasons but reach *opposite* conclusions then, again, this is something you will want the whole class to think about. This can be done effectively by engaging A and B with each other: 'A, B just gave the same reasons as you, but B thought 'not p' whereas you thought 'p'. Would you like to say anything about B's idea?' Sometimes, when children express themselves in, what *sounds* like, contradictions it may be that distinctions need to be drawn thereby exposing the apparent contradiction as an impostor. But whether or not what the children have said leads to contradictions or only *apparent* contradictions, these are in any case good opportunities for learning outcomes: if the children *reject* their positions because of contradictions or *refine* them with finely wrought distinctions.

Another key facilitation tool: *Concept-maps as tension detectors* – concept-maps (a 'boarding' device where you note just key concept-words on the board, linking them with lines and/or arrows to show relationships) are good to help a class (and you!) keep track of where the discussion has been. Above, I suggested that contradictions and tensions can be a great learning tool; well, the concept-map is a great tool for bringing out tensions and contradictions. However, I've seen concept-maps not used to their full potential in the classroom, where they are used merely to list all the children's different ideas about a concept. This is the right way to start, but to use concept maps to go further, they should be used to have the children see and then critically engage with each other over tensions that emerge from the listing part of concept-mapping. For example, the central concept might be 'mind'. One of the children says, 'Your mind is really your brain,' and the facilitator writes: 'mind = brain' coming off the central word (which is 'mind'). Someone else says, 'the mind is not physical, it's not *there*.' The facilitator writes, 'not physical,' steps back and takes a look at what's on the board, then says to the first child, 'Is the brain physical?' and he says, 'Yes.' The facilitator then draws a 'two-direction' arrow between these two ideas with a big question mark in the middle and she says, 'Is the mind physical?' to the whole class. This gets written up as an emergent task-question and the children go to talk-time.

3 **Critical thinking tool:** *Armchair philosophy* – there are some philosophers who think, for example, that the question as to whether time travel is possible or not is a logical issue not a scientific one. If time travel leads to a logical



contradiction (P and not P, see 'Contradictions and contraries' above) then, argue those philosophers, it is not and never will be possible. Yes, flight was considered impossible by some before it was achieved, but that was never a *logical* problem, just a question of technology. For these 'logical' philosophers, if something, such as time travel, leads to a contradiction, then time travel is as possible as that the sum 2 + 2 = 5 can be true. This kind of reasoning is known as *a priori* reasoning: reasoning for which no 'evidence from the world' is required, like maths or geometry.

Key facilitation tool: *Iffing the fact / either-or-the-if* – sometimes, empirical facts get in the way of a good discussion. 'Can we time travel?', 'They can't do brain transplants', 'Do all our cells get replaced every seven years?', 'It wouldn't be possible to get everyone to vote.' etc. These are just some of the factual issues that come up during philosophy sessions. Whatever you do, don't get drawn in! Instead, employ the conceptual techniques of the 'armchair philosopher': simply 'if' these facts like so: 'If everyone could vote and everyone voted that the Mona Lisa was the most beautiful painting, then would it be?' Or: 'If we were able to time travel then would you choose to do something other than you did the first time?' In some cases you may need to extend this technique a little; you may need to 'either-or-the-if': '*Let's think about it both ways*: if all our cells *are* replaced every seven years then would you be the same person after ten years?' Then 'go the other way': 'And if our cells are *not* all replaced, but only, say, 70% of them are, then would you be the same person after ten years?'

4 **Key critical thinking tool:** *Sets / classes* – a set or class is a group of objects or things. There is a branch of logic that organises objects and things into *sets*, *subsets* and *members*. For instance, there is the *set* of all things that are 'human beings'; the set of all things that are 'women' is a *subset* of the set of things that are 'human beings', and 'Charlotte Bronte' is a *member* of both the set of 'human beings' and the subset of 'women'. The set of all things that are 'humans beings' is a subset of the set of all things that are 'living things'.

Key facilitation tool: *Ropes and hoops* - asking children to categorise things is a useful way to explore a problem, to solve problems by drawing distinctions and see where there might be some grey area. For instance, one could invite a class to divide things up into two sets: 'real' and 'not real' (see 'Contradictories and contraries' above). You could use rope or hoops placed on the floor to form circles in which the children can place objects or cards/boards with words on. Using different sized ropes and hoops, the children could be encouraged to find subsets, and to overlap between sets. For instance, you could do a *same or different?* categorisation exercise on shapes with two hoops, one labeled 'same as a square' and the other, 'different from a square'. At first, one of the children might put a circle in the 'different' hoop and another square in the 'same' hoop, but then struggle to decide where to put a rectangle. Is it *same* or *different*? What the child needs is a new category, *similar*, but what the child might do - being without this third category - is to overlap the hoops and place the rectangle in the overlapped segment. Or, he or she may place the rectangle *between* the hoops.

Some maths-related discussion starters

What follows are three examples of how one can open up discussions about maths related topics. The first has a philosophical flavour to it, and the other two show how one can also enquire around maths topics without necessarily doing philosophy.

1: Exploring the nature of numbers

First, write this up on the board:

2	2
2	2

Starter question: How many numbers are here?
Task question: What is a number?

Then write this up:

0	0
0	0

And ask the same starter question.

Then write this up:

1 two

II . .

And ask the starter question again, always keeping in mind the main task question.

2: Exploring rules for single and double-digit numbers

- Begin by writing the number '22' on the board. Explain that this is the number 22.
- Write a '2' on an A4 piece of paper or similar size whiteboard and then write another '2' on another one.
- Place the two pieces of paper/boards at opposite sides of the room.

Task Question: Are the two '2's one number or two?

As an extension activity, have two children move the two '2's closer towards each other and ask the class the following:

Task question: At what point, if at all, do the two '2's become '22'?

3: Exploring identity '='

On separate pieces of A4 paper/whiteboards write the following:

2 + 2 = 4

Then ask if the 'sentence' makes sense.

Then, move the paper/boards around so it says:

4 = 2 + 2

And ask the class is this sentence makes sense.

Discuss what they think '=' means. At some point explain that it means 'the same as', if they haven't already explained this to each other. Then try this, also with A4 paper or whiteboards:

= = 'the same as'

Switch them round for some fun, asking if it still makes sense:

= 'the same as' =

'the same as' = =

= = 'the same as'

Finally, give the class some more examples to consider and discuss, such as:

Odysseus = Ulysses?
Cat = [insert a picture of a cat]?
[Definition of a square] = [insert picture of a square]?
You as a baby = you now?

References

https://en.wikipedia.org/wiki/Francesco_Redi
https://educationendowmentfoundation.org.uk/projects/philosophy-for-children/

To encourage good structure in thought and expression you need to learn good, structured questioning skills. Here are some places to go to find out more about the techniques described in this piece:

'If it, anchor it, open it up: A closed, guided questioning technique' by Worley, Peter (2015). https://books.google.co.uk/books?hl=en&lr=&id=bRS5CgAAQBAJ&oi=fnd&pg=PA131&dq=info:4ZsIECrIkuQJ:scholar.google.com&ots=O56wf4GvH9&sig=9NITLYzNf6kYrROqZipojIn5ZAc&redir_esc=y#v=onepage&q&f=false

The Socratic Handbook: Dialogue Methods For Philosophical Practice (ed. Michael Noah Weiss). Lit Verlag GmbH & Co. KG Wien.

Open thinking, closed questioning: http://www.ojs.unisa.edu.au/index.php/jps/article/view/1269

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